B.A/B.Sc. 5th Semester (Honours) Examination, 2019 (CBCS)

Subject : Mathematics

Paper : BMH5DSE23

(Boolean Algebra and Automata Theory)

Time: 3 Hours

Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meaning unless explained.

1. Answer *any ten* questions from the following: $2\times10=20$

(a) Write regular expressions for the following language:

The set of strings over alphabet $\{a, b, c\}$ containing at least one a and at least one b.

(b) Give an example of a regular expression for the languages of the automation.

(c) Show that the regular languages L are closed under following operation:
$\text{Min}(L) = \{w|\ w \text{ is in } L \text{ but no proper prefix of } w \text{ in } L\}$.

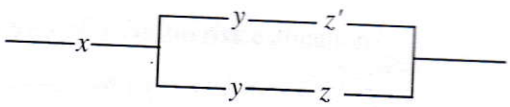
(d) Prove that every atom of a lattice with zero is join irreducible.

(e) Prove that any finite lattice is bounded.

(f) Give an example of a poset which has exactly one maximal element but does not have a greatest element.

(g) Prove that in a Boolean algebra B, the elements 0 and 1 are unique.

(h) Find the Boolean expression which represents the following circuit and simplify the expression if possible.



(i) In a Boolean algebra B, prove that $a + b = b$ implies $a.b = a$, for all $a, b \in B$.

(j) Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Let $x \leq y$ mean that x is a divisor of y. Find the maximal elements in the poset $(S \leq)$.

(k) In a Boolean algebra B, prove that $a + a' = 1$, and $aa' = 0$, for every $a \in B$, a' being the complement of a.

(l) Prove that the number of elements in a finite Boolean Algebra is a power of 2.

(m) Define a relation ρ on $\mathbb{C}$ by $(a + ib)\, \rho\, (c + id)$ if and only if $a \leq c$ and $b \leq d$ for $(a + ib), (c + id) \in \mathbb{C}$. Verify ρ is a partial order relation.

(n) Let $(S, \leq)$ be a poset. If $a, b \in S$ have a least upper bound then prove that it is unique.

(o) Let l_1 and l_2 be two distinct atoms in a Boolean algebra B. Then prove that $l_1.l_2 = 0$.

5×4=20

2. Answer *any four* questions from the following:

(a) (i) Let L and K be lattices with 0 and 1 and let $M = L \times M$. Show that there exists $a, b \in M$ such that $a \wedge b = (0,0)$ and $a \vee b = (1,1)$.

(ii) Define a lattice homomorphism and give an example of the same with justifications. 3+2=5

(b) (i) Suppose that L is any language, not necessary regular, whose alphabet is $\{0\}$. Prove that L^* is regular. 3+2=5

(ii) What is PDA (Push Down Automata)?

(c) Define CFG (Context-free Grammars). Consider the CFG G, defined by production $S \to a\, s\, b \mid b\, s\, a\, s \mid \in$. Prove that $L(G)$ is the set of all strings with equal numbers of a's and b's. 1+4=5

(d) (i) Let P and Q be finite ordered set and let $Q : P \to Q$ be a bijective map. Then prove that the following are equivalent:

(I) Q is an order-isomorphism.

(II) $x < y$ in P iff $Q(x) < Q(y)$ in Q.

(III) $x \prec y$ in P iff $Q(x) \prec Q(y)$ in Q.

(ii) Prove that the complement of an element in a Boolean Algebra is unique. 4+1=5

(e) (i) A Boolean function f is defined by $f(x, y, z) = xy + yz + zx$. Find the conjunctive normal form of $f(x, y, z)$. 4+1=5

(ii) Give an example of an atom in a Boolean algebra.

(f) Show that if P is a PAD (Push Down Automata) then there is a one-state PADP_1 such that $N(P_1) = N(P)$. 5

10×2=20

3. Answer *any two* questions from the following:

(a) (i) Show that if P is PDA, then there is a PDA P_2 with only two stack symbols such that $L(P_2) = L(P)$.

(ii) Convert the grammar

$S \to 0\, S\, 1 \mid A$

$A \to 0\, A\, 0 \mid S \mid \in$

to a PDA that accepts the same language by employ stack. 5+5=10

(b) (i) Is $F = \{A \subseteq \mathbb{N} \mid A \text{ finite}\}$ a sub lattices of $\wp(\mathbb{N})$? Is F bounded? Is $\wp(\mathbb{N})$ bounded?

(ii) In a Boolean algebra $(B, +, .)$ if $a + b = 0$, prove that $a.b' = 0$ and $a'.b = 0$. (3+2+2)+3=10

(c) (i) Show that the direct product of Boolean algebra is again a Boolean algebra.

(ii) Suppose that L is regular and that is recognized by a DFAM. Prove that there does not have exactly one accepting state. 5+5=10

(d) (i) Prove that the elements of an arbitrary lattice satisfy the following inequalities:

(I) $x \wedge (y \vee z) \geq (x \wedge y) \vee (x \wedge z)$

(II) $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$

$\forall\, x, y, z$ in the lattice.

(ii) Let $(\mathbb{R}, \leq)$ be the poset of all real numbers and let $A = \{x \in \mathbb{R} | x^3 < 3\}$. Is there an upper bound (or lower bound) of A? Justify your answer. (3+3)+4=10

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